Insurance Problems

A car insurance company sold 12,000 policies ($50,000 payout value) this year. The probability of an accident resulting in a claim for each policy is 0.002. The company charges $400 for each policy. Use the Poisson approximation to determine the following.

a) P(The company breaks even)

b) P(Company Profits $300,000 or more)

c) P(Company Loses $500,000 or more)

d) What value of λ would be used if the company charged $450 for each policy?

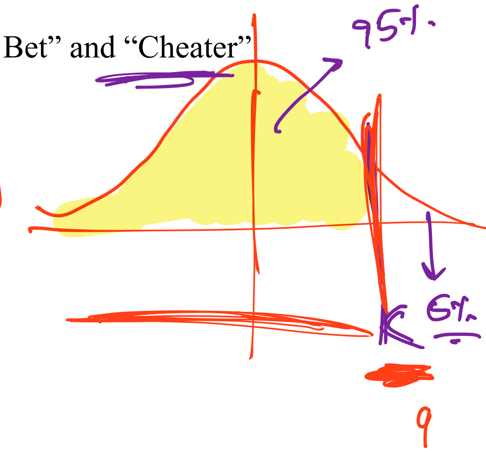
e) What value of λ would be used if the probability of an accident was 0.003?

$\boxed{\text{95-5\%}}$ Split – Binomial

$n = 13$ $p = 0.5$

Problem 1: We are going to toss a fair coin 13 times. Determine the "OK Bet" and "Cheater" Region for this.

$P(\bar{X} < K) \approx 0.95$ $[(0, 9), (10, 13)]$



If there were 7 Heads tossed, would you bet or not bet?

Ya

If the coin is fair is our above decision a theoretical error?

No

~~If so, will we eventually detect the error?~~

If the coin is a 70% coin is the above decision a theoretical error?

Ya

If so, will we eventually detect the error?

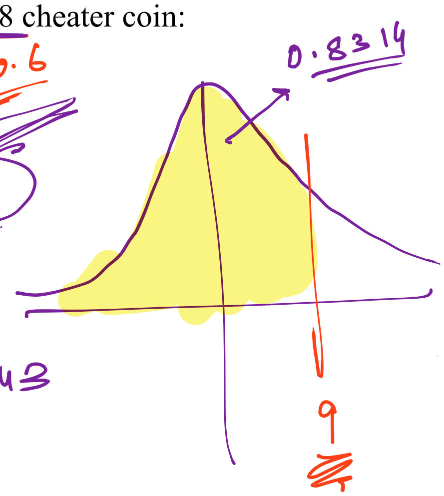
Yes

Determine the probability of detecting a p = 0.6, 0.7, and 0.8 cheater coin:

=> $n = 0.6$

$P(\bar{X} > 9) = 1 - 0.83 = 0.17$

$[10, 11, 12, 13]$

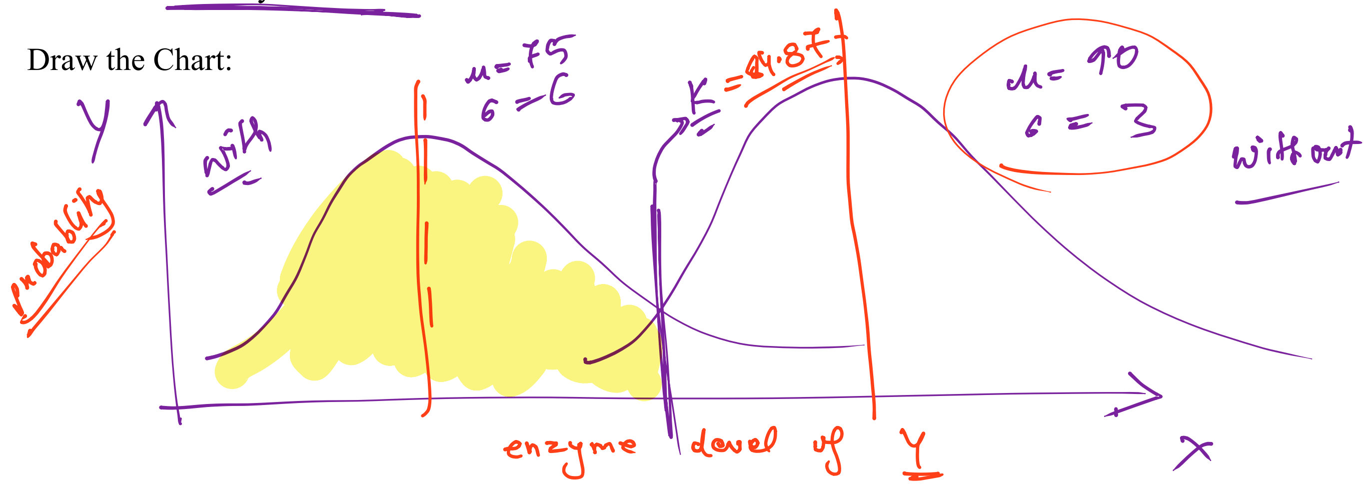


0.7 $P(\bar{X} > 9) = 1 - 0.5794 = 0.43$

0.8 $= 1 - \_\_\_ =$

For individuals with Condition X, the level of enzyme Y in the blood is normally distributed with a mean of 75 and a standard deviation of 6. For individuals without Condition X, enzyme Y levels are normally distributed with a mean of 90 and a standard deviation of 3.

Draw the Chart:



a) At what enzyme Y level should the "Tested Positive for Condition X" threshold start so that only 0.05 of people with Condition X would test negative?

$1.64 \rightarrow 0.9495$

$1.65 \rightarrow 0.9505$

$P(X < K) \approx 0.95$

$\Phi\left(\frac{K-\mu}{\sigma}\right) = 0.95$

$\Rightarrow \frac{K-\mu}{\sigma} = 1.645 \Rightarrow K = \sigma \times 1.645 + \mu$

$\Rightarrow K = 6 \times 1.645 + 75$

$K = 84.87$

0.95

1.645

b) What would be the probability of a false positive (an individual without Condition X tests positive)?

$\mu = 90$

$\sigma = 3$

$P(X < K) = \Phi\left(\frac{K-\mu}{\sigma}\right) = \Phi\left(\frac{84.87-90}{3}\right)$, with $K = 84.87$

$= \Phi(-1.71) = 0.0436$

$\Rightarrow 4.36\%$

c) A patient with Condition X has an enzyme Y level of 80. Will we properly diagnose that patient?

Yes

d) A patient with Condition X has an enzyme Y level of 85. Will we properly diagnose that patient?

No

e) A healthy patient has an enzyme Y level of 80. Will we properly diagnose that patient?

No

Solve on your own

Tire Warranty Problem

Problem 1: The life of tires of a certain tire company is known to be normally distributed with a mean of 50,000 miles and a standard deviation of 2500 miles.

$\frac{\bar{x} - \mu}{s}$

What is the probability that a randomly selected tire will last longer than 57,000 miles?

$\Rightarrow P(X > 57000) = P\left(Z > \frac{57000 - 50,000}{2500}\right) = P(Z > 2.8)$

$\Rightarrow 1 - \Phi(2.8) = 1 - 0.9974$

$\Rightarrow 0.0026$

Do you think a randomly selected tire will last longer than 57,000 miles?

$\Rightarrow$ NO

$\frac{52500 - \mu}{s}$

2 d.f. 0.26%

$P(47{,}500 < \mathit{Tire\ Life} < 52{,}500) =$

$\Rightarrow P(-1 < Z < 1) = \Phi(1) - \Phi(-1) = 0.6826$

Problem 2: The life of tires of a certain tire company is known to be normally distributed with a mean of 65,000 miles and a standard deviation of 3000 miles.

What is the probability that a randomly selected tire will last longer than 73,250 miles?

$\Rightarrow P(X > 73{,}250) = P\left(Z > \frac{73{,}2500 - 6500}{3000}\right)$

$= 1 - \Phi(\quad) = 0.003$

Do you think a randomly selected tire will last longer than 73,250 miles?

$\Rightarrow$ No

$P(40{,}500 < \mathit{Tire\ Life} < 52{,}500) = \Rightarrow$

$P\left(\frac{40{,}500 - 65000}{3000} < Z < \frac{52500 - 65000}{3000}\right)$